



Specification and Verification for

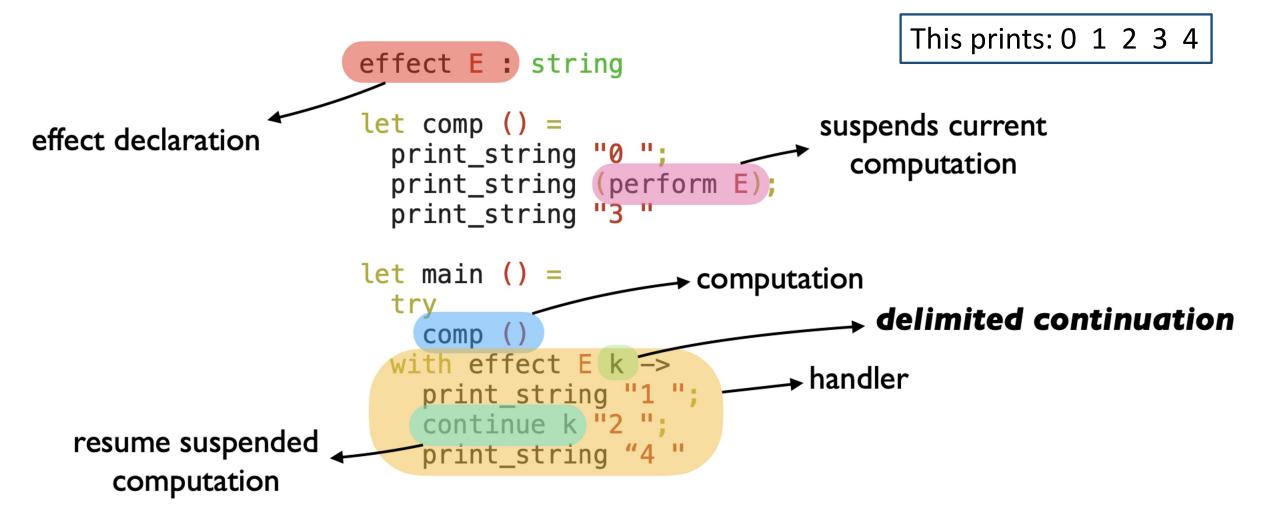
Unrestricted Algebraic Effects and Handling

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User-defined Effects and Handlers



Example taken from "Effect Handlers in Multicore OCaml" slides by KC Sivaramakrishnan.

Motivation Example

```
effect Label: int
1
  (* User-defined effect, which will be resumed with int values *)
2
3
  let callee () : int
4
  = let x = ref 0 in (* initialize x to zero *)
5
    let ret = perform Label in (* the handler has no access to x *)
6
7 	 x := !x + 1;
                 (* increment x from zero to one *)
 assert (!x = 1); (* x now contains one *)
8
9 ret + 2
                             (* return the resumed value + 2 *)
```

- Zero-shot handlers: abandon the continuation, just like exception handlers;
- One-shot handlers: resume the continuation once, the assertion on line 8 succeeds;
- Multi-shot handlers: resume the continuation more than once, the assertion on line 8 fails.

Motivation Example

```
1 effect Label: int
2 (* User-defined effect, which will be resumed with int values *)
3
4 let callee () : int
5 = let x = ref 0 in (* initialize x to zero *)
6 let ret = perform Label in (* the handler has no access to x *)
7 x := !x + 1; (* increment x from zero to one *)
8 assert (!x = 1); (* x now contains one *)
9 ret + 2 (* return the resumed value + 2 *)
```

Existing verification techniques:

- > multi-shot continuations + pure setting, e.g. [Song et al. 2022];
- heap manipulation + one-shot continuations, e.g. [de Vilhena and Pottier 2021];
- > multi-shot + heap-manipulation, under a restricted frame rule, e.g. [de Vilhena 2022].

Protocol Based Approach [de Vilhena and Pottier 2021]

- Hazel & Maze: Model client-handler interactions in the form of protocols
- Globally define the effects that clients perform and the replies they receive from handlers
- Global assumptions to provide explicit (or early) interpretation effects

 $ABORT \triangleq ! (()) \{ \text{True} \} ? y (y) \{ \text{False} \}$ $CXCHG \triangleq ! x x' (x') \{ \ell \mapsto x \} ? (x) \{ \ell \mapsto x' \}$ $AXCHG \triangleq ! x x' (x') \{ I x \} ? (x) \{ I x' \}$ $READWRITE \triangleq read \# ! x (()) \{ I x \} ? (x) \{ I x \}$ $+ write \# ! x x' (x') \{ I x \} ? (()) \{ I x \}$

Our Solution: Effectful Specification Logic (ESL)

- Fully Modular Per-Method Verification (no global assumption)
- Sequencing, φ_1 ; φ_2
- Uninterpreted relations for *unhandled effects* and unknown functions, *E(x, r)*
- Reducible *try-catch logic constructs*
- Normalization: compact each sequence of pre/post stages, via bi-abduction
- Use *re-summarization* (lemma) when handling recursive generated effects

$$(ESL) \qquad \varphi \quad ::= \quad \operatorname{req} P \mid \operatorname{ens}[r] \ Q \mid \varphi; \varphi \mid \varphi \lor \varphi \mid \exists x^*; \varphi \mid E(x, r) \mid f(x^*, r) \land \operatorname{try}[\delta](\varphi) \operatorname{catch} \mathcal{H}_{\Phi}$$

$$D, P, Q ::= \sigma \land \pi \qquad \qquad \sigma ::= emp \mid x \mapsto y \mid \sigma * \sigma \mid \dots$$

resu

We propose ESL

```
(ESL) \qquad \varphi \quad ::= \quad \operatorname{req} P \mid \operatorname{ens}[r] \ Q \mid \varphi; \varphi \mid \varphi \lor \varphi \mid \exists \ x^*; \varphi \mid \underbrace{E(x, r)} \mid f(x^*, r) \mid \operatorname{try}[\delta](\varphi) \operatorname{catch} \mathcal{H}_{\Phi}
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                               (* x now contains one *)
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   ret + 2
                               (* return the resumed value + 2 *)
9
```

$$\Rightarrow callee(r_c) = \exists x \cdot ens \ x \mapsto 0; \qquad // Line 5$$

$$\exists ret \cdot Label(ret); \qquad // Line 6$$

$$\exists z \cdot req \ x \mapsto z \wedge z+1=1 \ ens[r_c] \ x \mapsto z+1 \wedge r_c=(ret+2) \ // Lines \ 7-9$$

We propose ESL

 $(ESL) \qquad \varphi \quad ::= \quad \operatorname{req} P \mid \operatorname{ens}[r] \ Q \mid \varphi; \varphi \mid \varphi \lor \varphi \mid \exists \ x^*; \varphi \mid \underbrace{E(x, r)} \mid f(x^*, r) \mid \operatorname{try}[\delta](\varphi) \operatorname{catch} \mathcal{H}_{\Phi}$

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We propose ESL

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Try-Catch Reduction (Examples)

 $callee(r_c) = \exists x \cdot ens \ x \mapsto 0;$ // Line 5 // Line 6 $\exists ret \cdot Label(ret);$ $\exists z \cdot \operatorname{reg} x \mapsto z \wedge z + 1 = 1 \operatorname{ens}[r_c] x \mapsto z + 1 \wedge r_c = (\operatorname{ret} + 2) / / \operatorname{Lines} 7 - 9$

let zero_shot () : int $(* \ zero_shot(r_z) = \exists x ; \mathbf{ens}[r_z] \ x \mapsto 0 \land r_z = -1 \ *) \qquad (* \ one_shot(r_o) = \exists x ; \mathbf{ens}[r_o] \ x \mapsto 1 \land r_o = 5 \ *)$ = match callee () with effect Label k -> -1

let one_shot () : int = match callee () with | effect Label k -> resume k 3

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let zero_shot () : int (* $zero_shot(r_z) = \exists x ; ens[r_z] x \mapsto 0 \land r_z = -1 *)$ | (* $one_shot(r_o) = \exists x ; ens[r_o] x \mapsto 1 \land r_o = 5 *)$ = match callee () with effect Label k -> -1

let one_shot () : int = match callee () with | effect Label k -> resume k 3

```
let multi_shot () : int
(* multi\_shot(r_m) = req false *)
= match callee () with
| effect Label k ->
  let _ = resume k 4 in resume k 5
```

Try-Catch Reduction (Examples)

```
callee(r_c) = \exists x \cdot ens \ x \mapsto 0; \qquad // \text{ Line 5} 
\exists ret \cdot Label(ret); \qquad // \text{ Line 6} 
\exists z \cdot req \ x \mapsto z \land z+1=1 \ ens[r_c] \ x \mapsto z+1 \land r_c=(ret+2) \ // \text{ Lines 7-9}
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let zero_shot () : int (* zero_shot(r_z) = $\exists x$; ens[r_z] $x \mapsto 0 \land r_z = -1 *$) = match callee () with | effect Label k -> -1

```
let one_shot () : int
(* one_shot(r_o) = \exists x; ens[r_o] x \mapsto 1 \land r_o = 5 *)
= match callee () with
| effect Label k -> resume k 3
```

```
let multi_shot () : int
(* multi_shot(r<sub>m</sub>) = req false *)
= match callee () with
| effect Label k ->
let _ = resume k 4 in resume k 5
```

Intuition:

- > Explicit access to continuation
- > Modular verification:
 - try-catch reduction
 - normalization via bi-abduction

Try-Catch Reduction (Selected Rules)

- The base case: $(x \to \Phi_n) \in \mathcal{H}_{\Phi}$ $\overline{\operatorname{try}[\delta](\mathcal{N}[r])\operatorname{catch}\mathcal{H}_{\Phi} \rightsquigarrow \mathcal{N}[r]; \Phi_n[r/x]} \quad [\mathcal{R}\text{-Normal}]$
- When handling an effect, first reason about the behaviours of its continuation

$$\frac{\mathcal{E} = \mathcal{N}; \mathbf{E}(x, r) \qquad \mathbf{E} \in dom(\mathcal{H}_{\Phi}) \qquad \operatorname{try}[d](\theta) \operatorname{catch} \mathcal{H}_{\Phi} \rightsquigarrow \Phi}{\operatorname{try}[d](\mathcal{E}; \theta) \operatorname{catch} \mathcal{H}_{\Phi} \rightsquigarrow \operatorname{try}[d](\mathcal{E} \# \Phi) \operatorname{catch} \mathcal{H}_{\Phi}} \qquad [\mathcal{R}\text{-}Deep]$$

• Instantiate the high-order predicate k using the continuation's specification

 $\frac{\mathcal{E} = \mathcal{N}; \mathbf{E}(x, r) \quad (\mathbf{E}(y)k \to \Phi) \in \mathcal{H}_{\Phi} \quad \Phi' = \Phi[x/y, (\lambda(r, r_c) \to \Phi[r_c])/k]}{\operatorname{try}[\delta](\mathcal{E} \# \Phi[r_c]) \operatorname{catch} \mathcal{H}_{\Phi} \rightsquigarrow \mathcal{N}; \Phi'} \qquad [\mathcal{R}\text{-Eff-Handle}]$

Try-Catch Reduction (Selected Rules)

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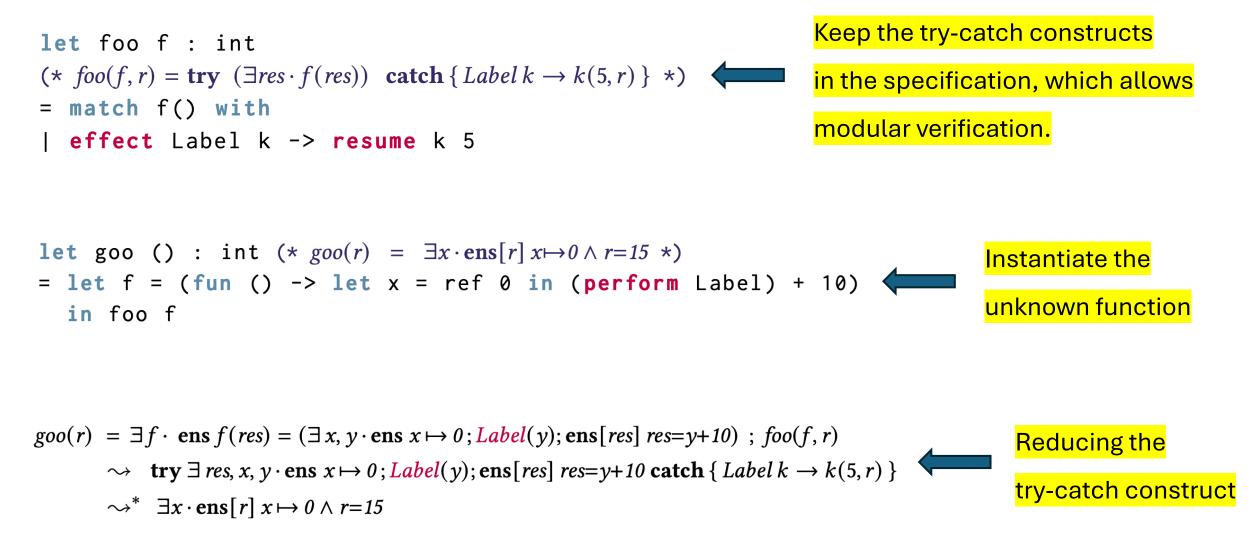
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Instantiate the high-order predicate k using the continuation's specification
$$= \mathcal{N} ; \mathbf{E}(x, r) \qquad (\mathbf{E}(y)k \rightarrow \Phi) \in \mathcal{H}_{\Phi} \qquad \Phi' = \Phi[x/y, (\lambda(r, r_c) \rightarrow \Phi[r_c])/k] \qquad [\mathcal{R}\text{-Eff-Handle}]$$

$$\mathbf{try}[\delta](\mathcal{E} \# \Phi[r_c]) \operatorname{catch} \mathcal{H}_{\Phi} \rightsquigarrow \mathcal{N} ; \Phi'$$
Binding the effect free continuation to k
$$= 14$$

•

3

Higher-Order Function meets Unresolved Try-Catch Construct



```
effect Flip : bool
1
2
    let tossN n
3
    (* tossN(n, res) = \exists r_0; ens n=1; Flip(r_0); ens[res] res=r_0 \lor
4
                     \exists r_1; \text{ ens } n > 1; Flip(r_1); \exists r_2; tossN(n-1, r_2); \text{ ens}[res] res=(r_1 \land r_2) *)
   = match n with
5
                                                                                                     2
                                                                                                                     3
    | 1 -> perform Flip
6
    | n -> let r1 = perform Flip in
7
            let r2 = tossN (n-1) in r1 && r2
8
                                                                                                         5
                                                                                                                 6
9
                                         Conjunct each Flip result
    let all results counter n
10
                                                                                                 n=1, counter = 2, res = 1
    (* all\_results(n, r) = \exists z; req counter \mapsto z \land n > 0 ens[r] counter \mapsto z + (2^{n+1} - 2) \land r = 1 *)
11
   = match tossN n with
12
                                                                                                 n=2, counter = 6, res =1
      | x \rightarrow if x then 1 else 0
13
      | effect Flip k ->
14
                                                                                                 n=3, counter=14, res =1
           counter := !counter + 1;
                                                      (* increase the counter *)
15
          let res1 = resume k true in
                                                       (* resume with true
                                                                                     *)
16
                                                                                                 ...
                                                       (* increase the counter *)
          counter := !counter + 1;
17
          let res2 = resume k false in
                                                       (* resume with false
                                                                                     *)
18
                                                        (* gather the results
          res1 + res2
                                                                                     *)
19
```

```
effect Flip : bool
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   let tossN n
3
   (* tossN(n, res) = \exists r_0; ens n=1; Flip(r_0); ens[res] res=r_0 \lor
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                     \exists r_1; ens n > 1; Flip(r_1); \exists r_2; tossN(n-1, r_2); ens[res] res=(r_1 \land r_2) *)
   = match n with
5
                                                                                                   2
                                                                                                                  3
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                                                                                                       5
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          counter := !counter + 1;
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                                                     (* resume with true
                                                                                   *)
16
                                                                                               ...
                                                     (* increase the counter *)
          counter := !counter + 1;
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          let res2 = resume k false in
                                                      (* resume with false
                                                                                   *)
18
                                                      (* gather the results
          res1 + res2
                                                                                   *)
19
```

Sum up how many back tracking branches leads to all true

```
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   = match n with
5
                                                                                                                    3
                                                                                                     2
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7
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8
                                                                                                         5
                                                                                                                6
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                                                                                                n=1, counter = 2, res = 1
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                                                                                                n=3, counter=14, res =1
                                                      (* increase the counter *)
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          let res1 = resume k true in
                                                      (* resume with true
                                                                                    *)
16
                                                      (* increase the counter *)
          counter := !counter + 1;
17
          let res2 = resume k false in
                                                      (* resume with false
                                                                                                 ..., counter = 2^{n+1}-2, res=1
                                                                                    *)
18
                                                       (* gather the results
          res1 + res2
                                                                                     *)
19
```

```
1 effect Flip : bool
```

```
try \exists res; tossN(n, res) # \exists r; ens[r] (acc\land res)\land r=1 \lor \neg (acc \land res) \land r=0 catch \mathcal{H}_{\Phi} \sqsubseteq \exists r; \Phi_{inv}(n, acc, r)
```

```
\Phi_{inv}(n, acc, r) = \exists w; \text{ req counter} \mapsto w \operatorname{ens}[r] \operatorname{counter} \mapsto w + (2^{n+1} - 2) \land (acc \land r = 1 \lor \neg acc \land r = 0)
```

```
7 | n -> let r1 = perform Flip in
          let r2 = tossN (n-1) in r1 \&\& r2
8
9
   let all results counter n
10
   (* all\_results(n, r) = \exists z; req counter \mapsto z \land n > 0 ens[r] counter \mapsto z + (2^{n+1} - 2) \land r = 1 *)
11
   = match tossN n with
12
     | x \rightarrow if x then 1 else 0
13
    | effect Flip k ->
14
                                  (* increase the counter *)
    counter := !counter + 1;
15
    let res1 = resume k true in (* resume with true
16
                                                                      *)
   counter := !counter + 1; (* increase the counter *)
17
  let res2 = resume k false in (* resume with false
                                                                      *)
18
                                              (* gather the results *)
    res1 + res2
19
```

1 **effect** Flip : bool

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try \exists res; tossN(n, res) # \exists r; ens[r] (acc \land res) \land r=1 \lor \neg (acc \land res) \land r=0 catch \mathcal{H}_{\Phi} \sqsubseteq \exists r; \Phi_{in\nu}(n, acc, r)
```

 $\Phi_{inv}(n, acc, r) = \exists w; \text{ req counter} \mapsto w \operatorname{ens}[r] \operatorname{counter} \mapsto w + (2^{n+1} - 2) \wedge (acc \wedge r = 1 \vee \neg acc \wedge r = 0)$

```
7 | n -> let r1 = perform Flip in
```

```
let r2 = tossN (n-1) in r1 && r2
```

```
8
9
```

```
10 let all_results counter n
```

```
11 (* all\_results(n, r) = \exists z ; req counter \mapsto z \land n > 0 ens[r] counter \mapsto z + (2^{n+1}-2) \land r=1 *)
```

```
12 = match tossN n with
```

```
13 | x \rightarrow if x then 1 else 0
```

```
14 | effect Flip k ->
```

```
counter := !counter + 1;
```

```
16 let res1 = resume k true in
```

```
17 counter := ! counter + 1;
```

```
18 let res2 = resume k false in
```

```
19 res1 + res2
```

```
• Proving via applying lemmas
```

- Lemmas are proved based on:
 - ✓ Try-catch reduction
 - ✓ Unfolding and rewriting (entailment rules)

Implementation and Evaluation

- 5K LoC on top of OCaml 5
- Benchmark programs with features: (Ind) proof is inductive, (MultiS)multi-shot handlers, (ImpureC) impure continuations, (HO) program is higher-order.

#	Program	Ind	MultiS	ImpureC	HO	LoC	LoS	Total(s)	AskZ3(s)
1	State monad	X	×	✓	X	126	16	8.54	6.21
2	Inductive sum		×	1	X	41	11	1.68	1.28
3	Flip-N (<u>Deep Ri</u> ght Rec) (Fig. 7)		1	\checkmark	X	39	10	2.09	1.52
4	Flip-N (Deep Left Rec)		1	\checkmark	X	45	13	2.03	1.53
5	Flip-N (Shallow Right Rec)	X	1	\checkmark	X	37	11	5.08	3.18
6	Flip-N (Shallow Left Rec)		1	\checkmark	X	64	23	6.75	4.26
7	McCarthy's amb operator (Fig. 25)	1	1	1	✓ ,	109	45	7.71	5.34
	Total	-	-	-	-	461	129	33.88	23.32

LoS/LoC < 30%

Summary

- ✓ **Scope**: Zero/one/multi-shots + impure continuations, deep/shallow handlers, left/right recursion
- ✓ Effectful Specification Logic: Staged specifications + unhandled effects + try-catch logic constructs
- ✓ **Hoare-style Verifier**: ML-like language + imperative higher-order + algebraic effects.
- ✓ **The Back-end Checker for ESL**: Normalization rules + reduction process of try-catch constructs.
- ✓ **Prototype (Multicore OCaml):** Proven correctness, report on experimental results, and case studies.

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✓ **Scope**: Zero/one/multi-shots + impure continuations, deep/shallow handlers, left/right recursion

- ✓ Effectful Specification Logic: Staged specifications + unhandled effects + try-catch logic constructs
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- ✓ **The Back-end Checker for ESL**: Normalization rules + reduction process of try-catch constructs.
- ✓ **Prototype (Multicore OCaml):** Proven correctness, report on experimental results, and case studies.

Take Away:

- 1) **Try not to assume,** for both HO functions and effects!
- 2) Staged logic + try-catch enable modular specifications without global assumptions (protocols).
- 3) Explicit access to the continuations, which can be composed as needed.

I am currently on the academic job market, looking for research positions! Thanks for listening!