

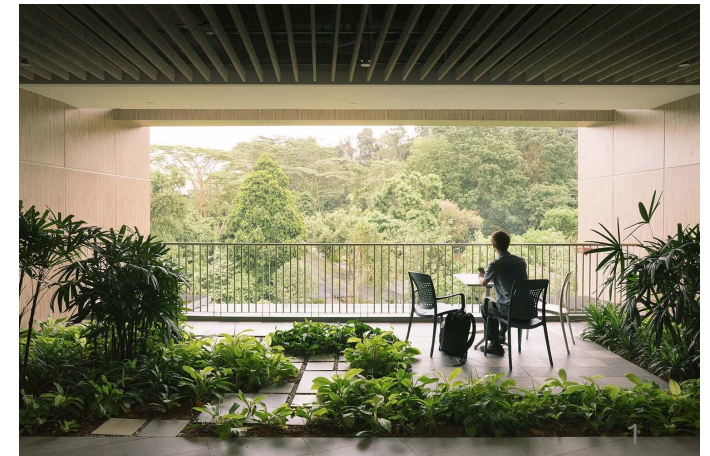
Staged Specification Logic

Higher-order Imperative Programs + Algebraic Effects

Yahui Song

Research Fellow @ National University of Singapore (NUS)

September 2024



My Research

- PhD (2018 Aug – 2023 May)

Thesis: Symbolic Temporal Verification Techniques with Extended Regular Expressions

Keywords: Modularly (Scalability), Expressive Specification, Hoare-style Verification

Applications {
Event-based reactive systems [ICFEM 2020]
Synchronous languages like Esterel [VMCAI 2021]
User-defined algebraic effects and handlers [APLAS 2022]
Real-time systems [TACAS 2023]

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- Research Fellow (2023 – now)

{ Temporal Property Guided Bug Detection and Repair [FSE 2024]

Staged Specification Logic:

Higher-order Imperative Programs [FM 2024]

Unrestricted Algebraic Effects and Handling [ICFP 2024]





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Staged Specification Logic for Verifying Higher-Order Imperative Programs

Darius Foo, [Yahui Song](#), Wei-Ngan Chin

Challenges with Effectful Higher-order Functions

- Existing (automated) verifier varies greatly:
 - Pure only: Dafny, WhyML, Cameleer
 - Type system (Rust) guarantees: Creusot, Prusti
 - Interactive: Iris, CFML, Pulse/Steel (F*)
- When supported, specifications are *imprecise*
- Is there a ***precise and general*** way to support effectful higher-order functions in ***automated program verifiers***?

Specification in Iris

Some clients may want to operate
only on certain kinds of lists

f must preserve the invariant

$$\forall P, Inv, f, xs, l. \left\{ \begin{array}{l} (\forall x, a', ys. \{P\ x * Inv\ ys\ a'\} \rightarrow f(x, a') \{r. Inv\ (x::ys)\ r\}) \\ * isList\ l\ xs * all\ P\ xs * Inv\ []\ a \end{array} \right\}$$

(Separation logic) Invariant
relating input to result

foldr should not change the list $foldr\ f\ a\ l$

$$\{r. isList\ l\ xs * Inv\ xs\ r\}$$

```
let rec foldr f a l =  
  match l with  
  | [] => a  
  | h :: t =>  
    f h (foldr f a t)
```

The Use of Abstract Properties

$$\forall P, Inv, f, xs, l. \left\{ \begin{array}{l} (\forall x, a', ys. \{P\ x * Inv\ ys\ a'\} \ f(x, a') \ \{r. Inv\ (x::ys)\ r\}) \\ * isList\ l\ xs * all\ P\ xs * Inv\ []\ a \\ foldr\ f\ a\ l \\ \{r. isList\ l\ xs * Inv\ xs\ r\} \end{array} \right\}$$

- foldr commits to an *abstraction* of f's behavior
- The abstraction may not be precise enough for a given client

The specification of `foldr` is higher-order in the sense that it involves nested Hoare triples (here in the precondition). The reason being that `foldr` takes a function f as argument, hence we can't specify `foldr` without having some knowledge or specification for the function f . Different clients may instantiate `foldr` with some very different functions, hence it can be hard to give a specification for f that is reasonable and general enough to support all these choices. In particular knowing when one has found a good and provable specification can be difficult in itself.

<https://iris-project.org/tutorial-pdfs/iris-lecture-notes.pdf> (page 32)

Difficult to handle cases like:

- Problem 1: Mutable list

```
let foldr_ex1 l = foldr (fun x r -> let v = !x
                                   in x := v+1; v+r) l 0
```

- Problem 2: Strengthened precondition

```
let foldr_ex2 l = foldr (fun x r -> assert(x+r>=0);x+r) l 0
```

- Problem 3: Exceptions/effects

```
let foldr_ex3 l = foldr (fun x r -> if x>=0 then x+r
                                   else raise Exc()) l 0
```

$$\forall P, Inv, f, xs, l. \left\{ \begin{array}{l} (\forall x, a', ys. \{P\ x * Inv\ ys\ a'\} \ f(x, a')\ \{r. Inv\ (x::ys)\ r\}) \\ * isList\ l\ xs * all\ P\ xs * Inv\ []\ a \end{array} \right\}$$

$$\quad foldr\ f\ a\ l$$

$$\{r. isList\ l\ xs * Inv\ xs\ r\}$$

Difficult to handle cases like:

- Problem 1: Mutable list

```
let foldr_ex1 l = foldr (fun x r -> let v = !x
                                   in x := v+1; v+r) l 0
```

- Problem 2: Strengthened precondition

```
let
```

- P

```
let foldr_ex2 l = foldr (fun x r -> if x > 0 then x+r
                                   else raise Exc()) l 0
```

$$\forall P, Inv, f, xs, l. \left\{ \begin{array}{l} (\forall x, a', ys. \{P\ x * Inv\ ys\ a'\} \rightarrow f(x, a') \{r. Inv\ (x::ys)\ r\}) \\ * isList\ l\ xs * all\ P\ xs * Inv\ []\ a \end{array} \right\} \\ foldr\ f\ a\ l \\ \{r. isList\ l\ xs * Inv\ xs\ r\}$$

We Propose “Staged Specification Logic”

Sequencing (Un)interpreted relations



$$\varphi ::= \mathbf{req} P \mid \mathbf{ens}[r] Q \mid \varphi; \varphi \mid f(x, r) \mid \exists y. \varphi \mid \varphi \vee \varphi$$
$$D, P, Q ::= \sigma \wedge \pi \qquad \sigma ::= emp \mid x \mapsto y \mid \sigma * \sigma \mid \dots$$

1. *Sequencing and uninterpreted relations*
2. *Recursive formulae*
3. *Re-summarization of recursion/lemmas*
4. *Compaction via bi-abduction*

$\Rightarrow$ *Defer abstraction until appropriate*

1. Sequencing and uninterpreted relations

$$\varphi ::= \mathbf{req} P \mid \mathbf{ens}[r] Q \mid \varphi; \varphi \mid f(x, r) \mid \exists y. \varphi \mid \varphi \vee \varphi$$

let hello f x y = x := !x + 1; let r = f y in let r2 = !x + r in y := r2; r2		<i>hello</i> (f, x, y, res) = $\exists a. \mathbf{req} x \mapsto a; \mathbf{ens} x \mapsto a+1;$ $\exists r. f(y, r);$ $\exists b. \mathbf{req} x \mapsto b * y \mapsto -;$ $\mathbf{ens} x \mapsto b * y \mapsto res \wedge res = b + r$
--	---	---

We can no longer assume anything about y at this point.

We also cannot assume anything about x!

2. Recursive formulae

$$\varphi ::= \mathbf{req} P \mid \mathbf{ens}[r] Q \mid \varphi; \varphi \mid f(x, r) \mid \exists y. \varphi \mid \varphi \vee \varphi$$

```
let rec foldr f a l =
  match l with
  | [] => a
  | h :: t =>
    f h (foldr f a t)
```



```
foldr(f, a, l, res) =
  ens l=[] ∧ res=a
  ∨ ∃ r, l1; ens l=x::l1;
    foldr(f, a, l1, r); f(x, r, res)
```

$$\forall P, Inv, f, xs, l. \left\{ \begin{array}{l} (\forall x, a', ys. \{P x * Inv ys a'\} f(x, a') \{r. Inv (x::ys) r\}) \\ * isList l xs * all P xs * Inv [] a \\ foldr f a l \\ \{r. isList l xs * Inv xs r\} \end{array} \right\}$$


```
foldr(f, a, l, res) =
  ∃ P, Inv, xs. req List(l, xs) * Inv([], a) ∧ all(P, xs)
  ∧ f(x, a', r) ⊆ (∃ ys. req Inv(ys, a') ∧ P(x); ens[r] Inv(x::ys, r));
  ens[res] List(l, xs) * Inv(xs, res)
```

$$\pi ::= \dots \mid \varphi \sqsubseteq \varphi$$

3. Re-summarization of recursion via lemmas

$$\varphi ::= \mathbf{req} P \mid \mathbf{ens}[r] Q \mid \varphi; \varphi \mid f(x, r) \mid \exists y. \varphi \mid \varphi \vee \varphi$$

```
let foldr_sum_state x xs init
```

```
= let g c t = x := !x + c; c + t in foldr g xs init
```

3. Re-summarization of recursion via lemmas

$$\varphi ::= \mathbf{req} P \mid \mathbf{ens}[r] Q \mid \varphi; \varphi \mid f(x, r) \mid \exists y. \varphi \mid \varphi \vee \varphi$$

```

let foldr_sum_state x xs init
foldr_sum_state(x, xs, init, res) =
   $\exists i, r. \mathbf{req} x \mapsto i; \mathbf{ens}[res] x \mapsto i+r \wedge res = r + init \wedge sum(xs, r)$ 
= let g c t = x := !x + c; c + t in foldr g xs init
  
```

$sum(li, res) =$
 $l = [] \wedge res = 0$
 $\vee \exists r, l_1. l = x :: l_1 \wedge sum(l_1, r) \wedge res = x + r$

$$\begin{aligned}
 foldr(f, a, l, res) = & \\
 & \mathbf{ens} l = [] \wedge res = a \\
 & \vee \exists r, l_1; \mathbf{ens} l = x :: l_1; \\
 & \quad foldr(f, a, l_1, r); f(x, r, res)
 \end{aligned}$$

$\forall x, xs, init, res. \quad foldr(g, xs, init, res)$
 $\sqsubseteq \exists i, r. \mathbf{req} x \mapsto i; \mathbf{ens} x \mapsto i+r \wedge res = r + init \wedge r = sum(xs)$

3. Re-summarization of recursion via lemmas

$$\begin{aligned}\varphi &::= \mathbf{req} P \mid \mathbf{ens}[r] Q \mid \varphi; \varphi \mid f(x, r) \mid \exists y. \varphi \mid \varphi \vee \varphi \\ D, P, Q &::= \sigma \wedge \pi \qquad \sigma ::= emp \mid x \mapsto y \mid \sigma * \sigma \mid \dots\end{aligned}$$

- Recovering abstraction: proving entailments

$$\begin{array}{c} \frac{}{x \mapsto i \vdash \exists i. x \mapsto i * emp} \text{SL} \quad \frac{}{x \mapsto i+r+h \wedge res=h+r+init \wedge r=sum(t)} \text{SL} \\ \frac{}{\vdash \exists r. x \mapsto i+r \wedge res=r+init \wedge r=sum(h::t)} \text{ENTAIL} \\ \frac{\exists i. \mathbf{req} x \mapsto i; \exists r, h, t. \mathbf{ens} x \mapsto i+r+h \wedge res=h+r+init \wedge r=sum(t) \wedge xs=h::t}{\sqsubseteq \exists i. \mathbf{req} x \mapsto i; \exists r. \mathbf{ens} x \mapsto i+r \wedge res=r+init \wedge r=sum(xs)} \text{NORMALIZE} \\ \frac{\exists r_1, h, t. \mathbf{ens} xs=h::t; \exists i, r. \mathbf{req} x \mapsto i; \mathbf{ens} x \mapsto i+r \wedge r_1=r+init \wedge r=sum(t); \exists b. \mathbf{req} x \mapsto b; \mathbf{ens} x \mapsto b+h \wedge res=h+r_1 \sqsubseteq \dots}{\exists r_1, h, t. \mathbf{ens} xs=h::t; \exists i, r. \mathbf{req} x \mapsto i; \mathbf{ens} x \mapsto i+r \wedge r_1=r+init \wedge r=sum(t); g(h, r_1, res) \sqsubseteq \dots} \text{UNFOLD} \\ \frac{(\dots \vee \exists r_1, h, t. \mathbf{ens} xs=h::t; \mathbf{foldr}(g, t, init, r_1); g(h, r_1, res)) \sqsubseteq \dots}{\forall x, xs, init, res. \mathbf{foldr}(g, xs, init, res) \sqsubseteq \exists i, r. \mathbf{req} x \mapsto i; \mathbf{ens} x \mapsto i+r \wedge res=r+init \wedge r=sum(xs)} \text{UNFOLD} \end{array}$$

Inductive Step



3. Re-summarization of recursion via lemmas

$$\varphi ::= \mathbf{req} P \mid \mathbf{ens}[r] Q \mid \varphi; \varphi \mid f(x, r) \mid \exists y. \varphi \mid \varphi \vee \varphi$$

$$D, P, Q ::= \sigma \wedge \pi \quad \sigma ::= emp \mid x \mapsto y \mid \sigma * \sigma \mid \dots$$

- Recovering abstraction: proving entailments

Bi-abduction

$$\frac{D_a * D_1 \vdash D_2 * D_f}{\mathbf{ens} D_1; \mathbf{req} D_2 \Longrightarrow \mathbf{req} D_a; \mathbf{ens} D_f}$$

$$\begin{array}{c} \frac{}{x \mapsto i \vdash \exists i. x \mapsto i * emp} \text{SL} \quad \frac{x \mapsto i+r+h \wedge res=h+r+init \wedge r=sum(h:t)}{\vdash \exists r. x \mapsto i+r \wedge res=r+init \wedge r=sum(h:t)} \\ \hline \exists i. \mathbf{req} x \mapsto i; \exists r, h, t. \mathbf{ens} x \mapsto i+r+h \wedge res=h+r+init \wedge r=sum(t) \wedge xs=h:t \\ \sqsubseteq \exists i. \mathbf{req} x \mapsto i; \exists r. \mathbf{ens} x \mapsto i+r \wedge res=r+init \wedge r=sum(xs) \\ \hline \exists r_1, h, t. \mathbf{ens} xs=h:t; \exists i, r. \mathbf{req} x \mapsto i; \mathbf{ens} x \mapsto i+r \wedge r_1=r+init \wedge r=sum(t); \\ \quad \exists b. \mathbf{req} x \mapsto b; \mathbf{ens} x \mapsto b+h \wedge res=h+r_1 \sqsubseteq \dots \\ \hline \exists r_1, h, t. \mathbf{ens} xs=h:t; \exists i, r. \mathbf{req} x \mapsto i; \mathbf{ens} x \mapsto i+r \wedge r_1=r+init \wedge r=sum(t); g(h, r_1, res) \sqsubseteq \dots \\ \hline (\dots \vee \exists r_1, h, t. \mathbf{ens} xs=h:t; foldr(g, t, init, r_1); g(h, r_1, res)) \sqsubseteq \dots \\ \hline \forall x, xs, init, res. foldr(g, xs, init, res) \\ \sqsubseteq \exists i, r. \mathbf{req} x \mapsto i; \mathbf{ens} x \mapsto i+r \wedge res=r+init \wedge r=sum(xs) \end{array}$$

ENTAIL

NORMALIZE

UNFOLD

REWRITE

UNFOLD

3. Re-summarization of recursion via lemmas

$$\begin{aligned}\varphi &::= \mathbf{req} P \mid \mathbf{ens}[r] Q \mid \varphi; \varphi \mid f(x, r) \mid \exists y. \varphi \mid \varphi \vee \varphi \\ D, P, Q &::= \sigma \wedge \pi \qquad \sigma ::= emp \mid x \mapsto y \mid \sigma * \sigma \mid \dots\end{aligned}$$

- Recovering abstraction: proving entailments

$$\begin{array}{c} \frac{\frac{\frac{}{x \mapsto i \vdash \exists i. x \mapsto i * emp} \text{SL} \quad \frac{x \mapsto i+r+h \wedge res=h+r+init \wedge r=sum(t)}{\vdash \exists r. x \mapsto i+r \wedge res=r+init \wedge r=sum(h::t)} \text{SL}}{\exists i. \mathbf{req} x \mapsto i; \exists r, h, t. \mathbf{ens} x \mapsto i+r+h \wedge res=h+r+init \wedge r=sum(t) \wedge xs=h::t} \text{ENTAIL}}{\frac{\frac{\frac{}{\vdash \exists i. \mathbf{req} x \mapsto i; \exists r. \mathbf{ens} x \mapsto i+r \wedge res=r+init \wedge r=sum(xs)} \text{NORMALIZE}}{\exists r_1, h, t. \mathbf{ens} xs=h::t; \exists i, r. \mathbf{req} x \mapsto i; \mathbf{ens} x \mapsto i+r \wedge r_1=r+init \wedge r=sum(t); \exists b. \mathbf{req} x \mapsto b; \mathbf{ens} x \mapsto b+h \wedge res=h+r_1} \subseteq \dots} \text{UNFOLD}}{\frac{\frac{\frac{}{\exists r_1, h, t. \mathbf{ens} xs=h::t; \exists i, r. \mathbf{req} x \mapsto i; \mathbf{ens} x \mapsto i+r \wedge r_1=r+init \wedge r=sum(t); g(h, r_1, res)} \subseteq \dots} \text{REWRITE}}{\frac{(\dots \vee \exists r_1, h, t. \mathbf{ens} xs=h::t; foldr(g, t, init, r_1); g(h, r_1, res)) \subseteq \dots}{\forall x, xs, init, res. \frac{foldr(g, xs, init, res)}{\subseteq \exists i, r. \mathbf{req} x \mapsto i; \mathbf{ens} x \mapsto i+r \wedge res=r+init \wedge r=sum(xs)} \text{UNFOLD}} \end{array}$$

Solutions with Staged Logic

- Problem 1: Mutable list

```
let foldr_ex1 l = foldr (fun x r -> let v = !x  
                                in x := v+1; v+r) l 0
```

$$\begin{aligned} foldr_ex1(l, res) \sqsubseteq & \exists xs. \mathbf{req} List(l, xs); \\ & \exists ys. \mathbf{ens} List(l, ys) \wedge mapinc(xs) = ys \wedge sum(xs) = res \end{aligned}$$

- Problem 2: Strengthened precondition

```
let foldr_ex2 l = foldr (fun x r -> assert(x+r>=0); x+r) l 0
```

$$foldr_ex2(l, res) \sqsubseteq \mathbf{req} allSPos(l); \mathbf{ens} sum(l) = res$$

- Problem 3: Exceptions/effects

```
let foldr_ex3 l = foldr (fun x r -> if x>=0 then x+r  
                                else raise Exc()) l 0
```

$$foldr_ex3(l, res) \sqsubseteq \mathbf{ens} allPos(l) \wedge sum(l) = res \vee (\mathbf{ens}[_] \neg allPos(l); Exc())$$

**More about
Effects later!**

Implementation & Evaluation

- 5K LoC on top of OCaml 5
- Reasonably low verification time
- Feasibility & increased expressiveness over existing systems

Benchmark	Heifer				Cameleer [21]			Prusti [27]		
	LoC	LoS	T	T_P	LoC	LoS	T	LoC	LoS	T
map	13	11	0.66	0.58	10	45	1.25	-	-	-
map_closure	18	7	1.06	0.77		✗		-	-	-
fold	23	12	1.06	0.87	21	48	8.08	-	-	-
fold_closure	23	12	1.25	0.89		✗		-	-	-
iter	11	4	0.40	0.32		✗		-	-	-
compose	3	1	0.11	0.09	2	6	0.05	-	-	-
compose_closure	23	4	0.44	0.32		✗		✗	-	-
closure [24]	27	5	0.37	0.27		✗		13	11	6.75
closure_list	7	1	0.15	0.09		✗		-	-	-
applyN	6	1	0.19	0.17	12	13	0.37	-	-	-
blameassgn [11]	14	6	0.31	0.28		✗		13	9	6.24
counter [16]	16	4	0.24	0.18		✗		11	7	6.37
lambda	13	5	0.25	0.22		✗		-	-	-
	197	73			45	112		37	27	

LoS/LoC = 0.37

= 2.49

= 0.73

inexpressible

incomparable

Summary

$$\varphi ::= \mathbf{req} P \mid \mathbf{ens}[r] Q \mid \varphi; \varphi \mid f(x, r) \mid \exists y. \varphi \mid \varphi \vee \varphi$$

- Staged logic for effectful higher-order programs
 1. *Sequencing and uninterpreted relations*
 2. *Recursive formulae*
 3. *Re-summarization* of recursion/lemmas
 4. *Compaction* via bi-abduction $\Rightarrow$ *Defer abstraction* until appropriate
- Heifer – a new automated verifier: <https://github.com/hipsleek/heifer>

*Higher-order Effectful Imperative Function Entailments and Reasoning



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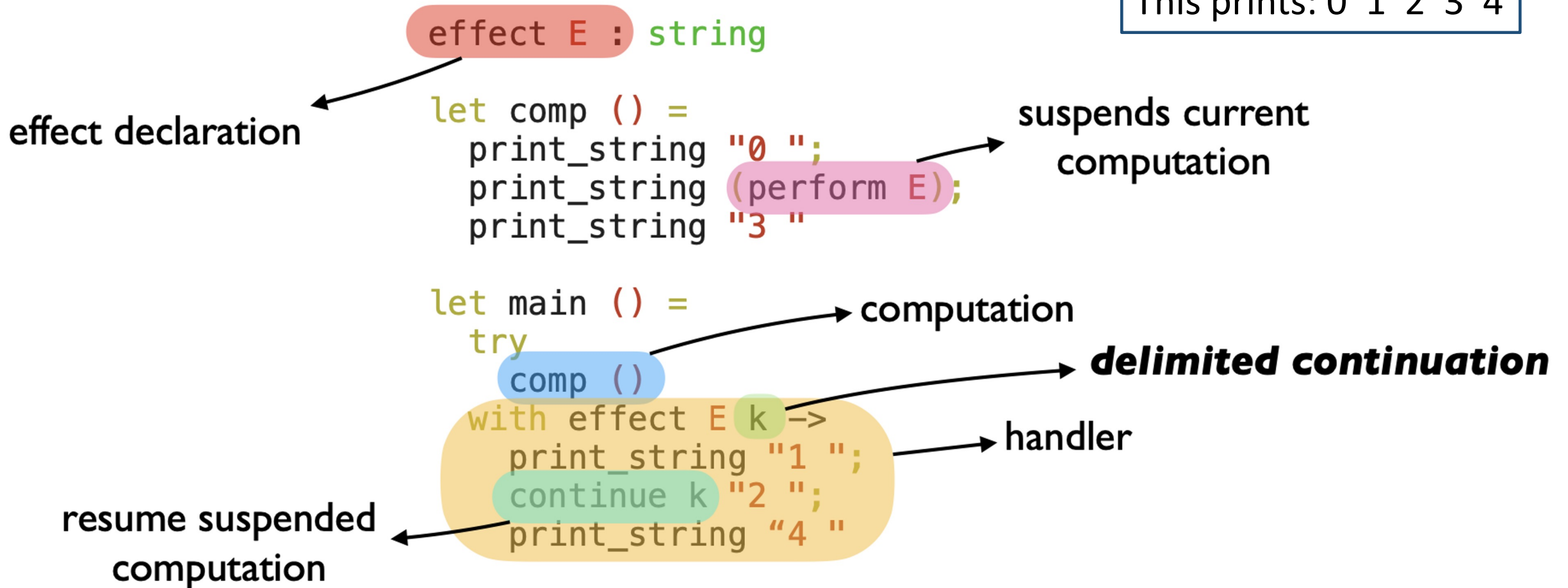
Specification and Verification for Unrestricted Algebraic Effects and Handling

Yahui Song, Darius Foo, Wei-Ngan Chin



User-defined Effects and Handlers

This prints: 0 1 2 3 4



Motivation Example

```
1  effect Label: int
2  (* User-defined effect, which will be resumed with int values *)
3
4  let callee () : int
5  = let x = ref 0 in                (* initialize x to zero *)
6    let ret = perform Label in      (* the handler has no access to x *)
7    x := !x + 1;                    (* increment x from zero to one *)
8    assert (!x = 1);                (* x now contains one *)
9    ret + 2                         (* return the resumed value + 2 *)
```

- Zero-shot handlers: abandon the continuation, just like exception handlers;
- One-shot handlers: resume the continuation once, the assertion on line 8 succeeds;
- Multi-shot handlers: resume the continuation more than once, the assertion on line 8 fails.

Motivation Example

```
1  effect Label: int
2  (* User-defined effect, which will be resumed with int values *)
3
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8    assert (!x = 1); (* x now contains one *)
9    ret + 2 (* return the resumed value + 2 *)
```

Existing verification techniques:

- multi-shot continuations + pure setting, e.g. [Song et al. 2022];
- heap manipulation + one-shot continuations, e.g. [de Vilhena and Pottier 2021];
- multi-shot + heap-manipulation, under a restricted frame rule, e.g. [de Vilhena 2022].

Protocol Based Approach [de Vilhena and Pottier 2021]

- Hazel & Maze: Model client-handler interactions in the form of protocols
- Globally define the effects that clients perform and the replies they receive from handlers
- Global assumptions to provide explicit (or early) interpretation effects

$$ABORT \triangleq ! \ () \{ \text{True} \}. ? \ y \ (y) \{ \text{False} \}$$

$$CXCHG \triangleq ! \ x \ x' \ (x') \{ \ell \mapsto x \}. ? \ (x) \{ \ell \mapsto x' \}$$

$$AXCHG \triangleq ! \ x \ x' \ (x') \{ I \ x \}. ? \ (x) \{ I \ x' \}$$

$$\begin{aligned} READWRITE \triangleq & \quad read \ \# \ ! \ x \ () \{ I \ x \}. ? \ (x) \{ I \ x \} \\ & + \ write \ \# \ ! \ x \ x' \ (x') \{ I \ x \}. ? \ () \{ I \ x' \} \end{aligned}$$

Motivation Example

```
1  effect Label: int
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Specification and Verification for Unrestricted Algebraic Effects and Handling

Yahui Song, Darius Foo, Wei-Ngan Chin

4th Sep @ ICFP 2024, Milan, Italy



Our Solution: Effectful Specification Logic (ESL)

- Fully modular per-method verification (no global assumption)
- Sequencing, $\varphi_1 ; \varphi_2$
- Uninterpreted relations for *unhandled effects* and unknown functions, $E(x, r)$
- Reducible *try-catch logic constructs*
- *Normalization: compact* each sequence of pre/post stages, via bi-abduction
- Use *re-summarization* (lemma) when handling recursive generated effects

result

input

$$(ESL) \quad \varphi ::= \text{req } P \mid \text{ens}[r] Q \mid \varphi; \varphi \mid \varphi \vee \varphi \mid \exists x^*; \varphi \mid$$

$$E(x, r) \mid f(x^*, r) \mid \text{try}[\delta](\varphi) \text{ catch } \mathcal{H}_\Phi$$

$$D, P, Q ::= \sigma \wedge \pi$$

$$\sigma ::= \text{emp} \mid x \mapsto y \mid \sigma * \sigma \mid \dots$$

We propose ESL

(ESL) $\varphi ::= \text{req } P \mid \text{ens}[r] Q \mid \varphi; \varphi \mid \varphi \vee \varphi \mid \exists x^*; \varphi \mid$
 $\text{E}(x, r) \mid f(x^*, r) \mid \text{try}[\delta](\varphi) \text{ catch } \mathcal{H}_\Phi$

```
1  effect Label: int
2  (* User-defined effect, which will be resumed with int values *)
3
4  let callee () : int
5  = let x = ref 0 in (* initialize x to zero *)
6    let ret = perform Label in (* the handler has no access to x *)
7    x := !x + 1; (* increment x from zero to one *)
8    assert (!x = 1); (* x now contains one *)
9    ret + 2 (* return the resumed value + 2 *)
```

➡ $\text{callee}(r_c) = \exists x \cdot \text{ens } x \mapsto 0; \quad // \text{ Line 5}$
 $\exists \text{ret} \cdot \text{Label}(\text{ret}); \quad // \text{ Line 6}$
 $\exists z \cdot \text{req } x \mapsto z \wedge z+1=1 \text{ ens}[r_c] x \mapsto z+1 \wedge r_c=(\text{ret}+2) // \text{ Lines 7-9}$

We propose ESL

(ESL) $\varphi ::= \text{req } P \mid \text{ens}[r] Q \mid \varphi; \varphi \mid \varphi \vee \varphi \mid \exists x^*; \varphi \mid$
 $\text{E}(x, r) \mid f(x^*, r) \mid \text{try}[\delta](\varphi) \text{ catch } \mathcal{H}_\Phi$

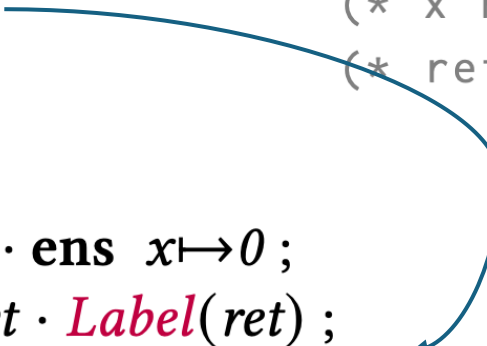
```

1  effect Label: int
2  (* User-defined effect, which will be resumed with int values *)
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4  let callee () : int
5  = let x = ref 0 in (* initi
6    let ret = perform Label in (* the h
7    x := !x + 1; (* incre
8    assert (!x = 1); (* x now
9    ret + 2 (* retur

```

Bi-abduction:

$\exists z; \text{req } x \rightarrow z;$
 $\text{ens } x \rightarrow z+1;$
 $\exists b; \text{req } x \rightarrow b \wedge b=1$


 $\text{callee}(r_c) = \exists x \cdot \text{ens } x \mapsto 0; \quad // \text{ Line 5}$
 $\exists \text{ret} \cdot \text{Label}(\text{ret}); \quad // \text{ Line 6}$
 $\exists z \cdot \text{req } x \mapsto z \wedge z+1=1 \text{ ens}[r_c] x \mapsto z+1 \wedge r_c = (\text{ret}+2) // \text{ Lines 7-9}$

We propose ESL

(ESL) $\varphi ::= \text{req } P \mid \text{ens}[r] Q \mid \varphi; \varphi \mid \varphi \vee \varphi \mid \exists x^*; \varphi \mid$
 $\text{E}(x, r) \mid f(x^*, r) \mid \text{try}[\delta](\varphi) \text{ catch } \mathcal{H}_\Phi$

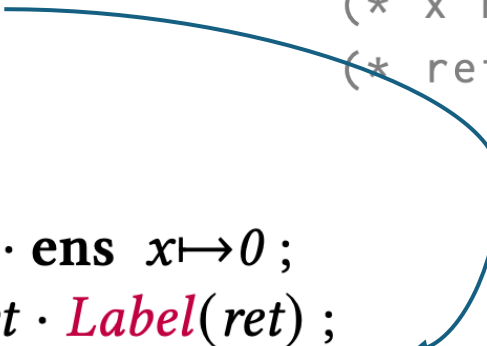
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8    assert (!x = 1); (* x now
9    ret + 2 (* return the resumed value + z *)

```

Bi-abduction:

$\exists z; \text{req } x \rightarrow z \wedge z+1=1;$
 $\text{ens } x \rightarrow z+1$


 $\text{callee}(r_c) = \exists x \cdot \text{ens } x \mapsto 0; \quad // \text{ Line 5}$
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Try-Catch Reduction (Examples)

$callee(r_c) = \exists x \cdot \mathbf{ens} \ x \mapsto 0 ;$ // Line 5
 $\exists ret \cdot \textcolor{red}{Label}(ret) ;$ // Line 6
 $\exists z \cdot \mathbf{req} \ x \mapsto z \wedge \textcolor{blue}{z+1=1} \ \mathbf{ens}[r_c] \ x \mapsto z+1 \wedge r_c = (ret+2)$ // Lines 7-9

```
let zero_shot () : int
(* zero_shot(rz) =  $\exists x ; \mathbf{ens}[r_z] \ x \mapsto 0 \wedge r_z = -1$  *)
= match callee () with
| effect Label k -> -1
```

```
let one_shot () : int
(* one_shot(ro) =  $\exists x ; \mathbf{ens}[r_o] \ x \mapsto 1 \wedge r_o = 5$  *)
= match callee () with
| effect Label k -> resume k 3
```

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= match callee () with
| effect Label k -> resume k 3
```

```
let multi_shot () : int
(* multi_shot( $r_m$ ) = req false *)
= match callee () with
| effect Label k ->
  let _ = resume k 4 in resume k 5
```

Try-Catch Reduction (Examples)

$callee(r_c) = \exists x \cdot \mathbf{ens} \ x \mapsto 0 ;$ // Line 5
 $\exists ret \cdot \textit{Label}(ret) ;$ // Line 6
 $\exists z \cdot \mathbf{req} \ x \mapsto z \wedge \textcolor{blue}{z+1=1} \ \mathbf{ens}[r_c] \ x \mapsto z+1 \wedge r_c = (ret+2)$ // Lines 7-9

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```

Intuition:

- Explicit access to continuation
- Modular verification:
 - try-catch reduction
 - normalization via bi-abduction

Try-Catch Reduction (Selected Rules)

- The base case:

$$\frac{(x \rightarrow \Phi_n) \in \mathcal{H}_\Phi}{\text{try}[\delta](\mathcal{N}[r]) \text{ catch } \mathcal{H}_\Phi \rightsquigarrow \mathcal{N}[r] ; \Phi_n[r/x]} [\mathcal{R}\text{-Normal}]$$

- When handling an effect, first reason about the behaviours of its continuation

$$\frac{\mathcal{E} = \mathcal{N} ; E(x, r) \quad E \in \text{dom}(\mathcal{H}_\Phi) \quad \text{try}[d](\theta) \text{ catch } \mathcal{H}_\Phi \rightsquigarrow \Phi}{\text{try}[d](\mathcal{E} ; \theta) \text{ catch } \mathcal{H}_\Phi \rightsquigarrow \text{try}[d](\mathcal{E} \# \Phi) \text{ catch } \mathcal{H}_\Phi} [\mathcal{R}\text{-Deep}]$$

- Instantiate the high-order predicate k using the continuation's specification

$$\frac{\mathcal{E} = \mathcal{N} ; E(x, r) \quad (E(y)k \rightarrow \Phi) \in \mathcal{H}_\Phi \quad \Phi' = \Phi[x/y, (\lambda(r, r_c) \rightarrow \Phi[r_c])/k]}{\text{try}[\delta](\mathcal{E} \# \Phi[r_c]) \text{ catch } \mathcal{H}_\Phi \rightsquigarrow \mathcal{N} ; \Phi'} [\mathcal{R}\text{-Eff-Handle}]$$

Try-Catch Reduction (Selected Rules)

- The base case:

$$\frac{(x \rightarrow \Phi_n) \in \mathcal{H}_\Phi}{\text{try}[\delta](\mathcal{N}[r]) \text{ catch } \mathcal{H}_\Phi \rightsquigarrow \mathcal{N}[r]; \Phi_n[r/x]} \quad [\mathcal{R}\text{-Normal}]$$

- When handling an effect, first reason about the behaviours of its continuation

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effect-free (wrt $\mathcal{H}_\Phi$) after #

- Instantiate the high-order predicate k using the continuation's specification

$$\frac{\mathcal{E} = \mathcal{N}; \textcolor{red}{E}(x, r) \quad (\textcolor{red}{E}(y)k \rightarrow \Phi) \in \mathcal{H}_\Phi \quad \Phi' = \Phi[x/y, (\lambda(r, r_c) \rightarrow \Phi[r_c])/k]}{\text{try}[\delta](\mathcal{E} \# \Phi[r_c]) \text{ catch } \mathcal{H}_\Phi \rightsquigarrow \mathcal{N}; \Phi'} \quad [\mathcal{R}\text{-Eff-Handle}]$$

Binding the effect free continuation to k

Higher-Order Function meets Unresolved Try-Catch Construct

```
let foo f : int
(* foo(f, r) = try (∃res. f(res)) catch { Label k → k(5, r) } *)
= match f() with
| effect Label k -> resume k 5
```

Keep the try-catch constructs
in the specification, which allows
modular verification.

```
let goo () : int (* goo(r) = ∃x. ens[r] x ↦ 0 ∧ r=15 *)
= let f = (fun () -> let x = ref 0 in (perform Label) + 10)
  in foo f
```

Instantiate the
unknown function

$$\begin{aligned} \text{goo}(r) &= \exists f. \text{ens } f(\text{res}) = (\exists x, y. \text{ens } x \mapsto 0; \text{Label}(y); \text{ens}[\text{res}] \text{res} = y + 10) ; \text{foo}(f, r) \\ &\rightsquigarrow \text{try } \exists \text{res}, x, y. \text{ens } x \mapsto 0; \text{Label}(y); \text{ens}[\text{res}] \text{res} = y + 10 \text{ catch } \{ \text{Label } k \rightarrow k(5, r) \} \\ &\rightsquigarrow^* \exists x. \text{ens}[r] x \mapsto 0 \wedge r = 15 \end{aligned}$$

Reducing the
try-catch construct

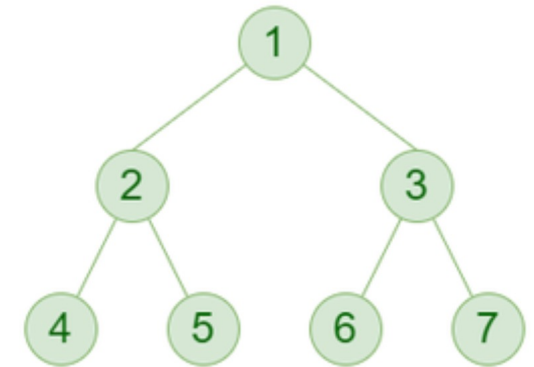
Inductive Proofs via Lemmas

```

1  effect Flip : bool
2
3  let tossN n
4  (* tossN(n, res) =  $\exists r_0$ ; ens  $n=1$ ; Flip( $r_0$ ); ens[res]  $res=r_0 \vee$ 
    $\exists r_1$ ; ens  $n>1$ ; Flip( $r_1$ );  $\exists r_2$ ; tossN( $n-1, r_2$ ); ens[res]  $res=(r_1 \wedge r_2)$  *)
5  = match n with
6  | 1 -> perform Flip
7  | n -> let r1 = perform Flip in
8         let r2 = tossN (n-1) in r1 && r2
9
10 let all_results counter n
11 (* all_results(n, r) =  $\exists z$ ; req counter  $\mapsto z \wedge n>0$  ens[r] counter  $\mapsto z+(2^{n+1}-2) \wedge r=1$  *)
12 = match tossN n with
13 | x -> if x then 1 else 0
14 | effect Flip k ->
15     counter := !counter + 1;          (* increase the counter *)
16     let res1 = resume k true in      (* resume with true *)
17     counter := !counter + 1;          (* increase the counter *)
18     let res2 = resume k false in    (* resume with false *)
19     res1 + res2                      (* gather the results *)

```

Conjunct each Flip result



$n=1$, counter = 2, res = 1

$n=2$, counter = 6, res = 1

$n=3$, counter=14, res = 1

...

... , counter = $2^{n+1}-2$, res=1

Inductive Proofs via Lemmas

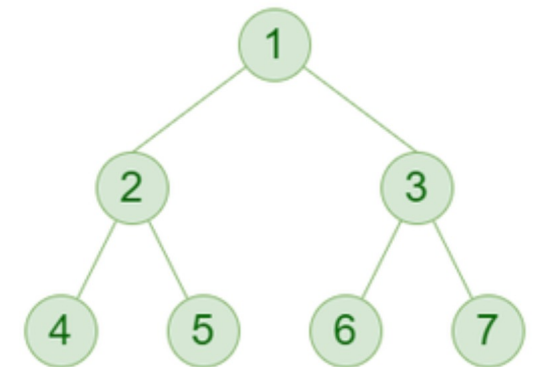
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Conjunct each Flip result

Sum up how many back tracking branches leads to all true



$n=1$, counter = 2, res = 1

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Inductive Proofs via Lemmas

```
1 effect Flip : bool
```

```
try  $\exists res; \text{tossN}(n, res) \# \exists r; \text{ens}[r] (acc \wedge res) \wedge r=1 \vee \neg(acc \wedge res) \wedge r=0$  catch  $\mathcal{H}_\Phi \sqsubseteq \exists r; \Phi_{inv}(n, acc, r)$ 
```

$$\Phi_{inv}(n, acc, r) = \exists w; \text{req } counter \mapsto w \text{ ens}[r] counter \mapsto w + (2^{n+1} - 2) \wedge (acc \wedge r=1 \vee \neg acc \wedge r=0)$$

```
7 | n -> let r1 = perform Flip in
8       let r2 = tossN (n-1) in r1 && r2
9
10 let all_results counter n
11   (* all_results(n, r) =  $\exists z; \text{req } counter \mapsto z \wedge n > 0 \text{ ens}[r] counter \mapsto z + (2^{n+1} - 2) \wedge r=1$  *)
12   = match tossN n with
13     | x -> if x then 1 else 0
14     | effect Flip k ->
15       counter := !counter + 1;          (* increase the counter *)
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```

- Proving via applying lemmas
- Lemmas are proved based on:
 - ✓ Try-catch reduction
 - ✓ Unfolding and rewriting (entailment rules)

Implementation and Evaluation

- 5K LoC on top of OCaml 5
- Benchmark programs with features: (Ind) proof is inductive, (MultiS) multi-shot handlers, (ImpureC) impure continuations, (HO) program is higher-order.

#	Program	Ind	MultiS	ImpureC	HO	LoC	LoS	Total(s)	AskZ3(s)
1	State monad	✗	✗	✓	✗	126	16	8.54	6.21
2	Inductive sum	✓	✗	✓	✗	41	11	1.68	1.28
3	Flip-N (Deep Right Rec) (Fig. 7)	✓	✓	✓	✗	39	10	2.09	1.52
4	Flip-N (Deep Left Rec)	✓	✓	✓	✗	45	13	2.03	1.53
5	Flip-N (Shallow Right Rec)	✗	✓	✓	✗	37	11	5.08	3.18
6	Flip-N (Shallow Left Rec)	✓	✓	✓	✗	64	23	6.75	4.26
7	McCarthy's amb operator (Fig. 25)	✓	✓	✓	✓	109	45	7.71	5.34
	Total	-	-	-	-	461	129	33.88	23.32

LoS/LoC < 30%

Summary

- ✓ **Scope:** Zero/one/multi-shots + impure continuations, deep/shallow handlers, left/right recursion
- ✓ **Effectful Specification Logic:** Staged specifications + unhandled effects + try-catch logic constructs
- ✓ **Hoare-style Verifier:** ML-like language + imperative higher-order + algebraic effects.
- ✓ **The Back-end Checker for ESL:** Normalization rules + reduction process of try-catch constructs.
- ✓ **Prototype (Multicore OCaml):** Proven correctness, report on experimental results, and case studies.

Summary

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- ✓ **The Back-end Checker for ESL:** Normalization rules + reduction process of try-catch constructs.
- ✓ **Prototype (Multicore OCaml):** Proven correctness, report on experimental results, and case studies.

Take Away:

Thanks!

- 1) **Try not to assume**, for both HO functions and effects!
- 2) Modular specifications without global assumptions: try-catch constructs.
- 3) Explicit access to the continuations, which can be composed as needed.

My Research

- PhD (2018 Aug – 2023 May)

Thesis: Symbolic Temporal Verification Techniques with Extended Regular Expressions

Keywords: Modularly (Scalability), Expressive Specification, Hoare-style Verification

Applications {
Event-based reactive systems [ICFEM 2020]
Synchronous languages like Esterel [VMCAI 2021]
User-defined algebraic effects and handlers [APLAS 2022]
Real-time systems [TACAS 2023]

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{ Temporal Property Guided Bug Detection and Repair [FSE 2024]
Staged Specification Logic [FM 2024] [ICFP 2024]



Future

➤ Staged Specification Logic

- 1) Summary/Lemma Inference;
- 2) Non-terminating/liveness behaviours of effects handling;
- 3) Apply staged specification to delimited control operators, e.g., shift/reset;
- 4) Combine staged specification with type system.

➤ Temporal Logic Based Bug Finding and Repair

- 1) Least Fixpoint & Greatest Fixpoint defined analysis : CTL + Datalog;
- 2) Liveness Checking: Termination + Safety checking + Fairness Assumption;
- 3) Temporal Logic Augmented LLM.

➤ ...

• PhD (2018 Aug – 2023 May)

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